Superconductivity & Quantum Coherence **Lent 2012**
Examples Sheet 2

1. Vortex Dynamics

Discuss the dynamics of vortices in a type II superconductor. Sketch the form of the hysteresis curve of magnetization vs applied magnetic field in the presence of vortex pinning centres. Compare and contrast the hysteresis curve with that of a typical ferromagnetic material.

2. Weak Junction

A resistively shunted Josephson junction is driven by a DC voltage V_0 plus an AC voltage of frequency f. Obtain an expression for the time dependence of the total current through the circuit in terms of the relevant circuit parameters and show that a DC Josephson effect appears at values of V_0 equal to integral multiples of f times the quantum of flux $\phi_0=h/2e$. Sketch the DC I-V characteristics of the circuit.

3. Aspects of Superfluidity

Write short notes on the following three topics, highlighting the relevant physics:

(a) the two-fluid model and the fountain effect;
(b) quantization of circulation and vortices;
(c) elementary excitations and the Landau critical velocity.

4. Bose-Einstein Condensation and ^{4}He

a) Show that BEC is possible for 3D free bosons, and for bosons in a 2D or 3D harmonic trapping potential---but not for 1D or 2D free bosons, or for a 1D harmonic trapping potential.
b) Determine the BEC transition temperature T_c expected for ^{4}He (density $\rho = 145$ kg/m^3) if we neglect interactions. [Answer: 3.1 K.]
c) Show that (in 3D) T_c is roughly the temperature where the thermal de Broglie wavelength reaches the interatomic spacing.

5. Coherent States

a) Show that two coherent states $|\alpha\rangle$ and $|\beta\rangle$ of the harmonic oscillator have an overlap

$$|\langle\alpha \mid \beta\rangle|^2 = e^{-|\alpha-\beta|^2} .$$

This means that no two coherent states are perfectly orthogonal, but their overlap rapidly diminishes with increasing $|\alpha - \beta| > 1$.

b) Coherent states are eigenstates of the annihilation operator. Do eigenstates of the creation operator exist and if so, what are their decomposition into the occupation number states?

6. Superconductivity and the Higgs Mechanism

Phonons are the Goldstone bosons of the superfluid order parameter, and as expected from Goldstone's theorem they are gapless, i.e., $\omega \to 0$ in the long wavelength limit $k \to 0$.

What are the equivalent excitations in a superconductor? What is their frequency as $k \to 0$? What is the status of Goldstone's theorem in this case?

[No detailed mathematical analysis is required. This is a deep conceptual problem connected with the Anderson-Higgs mechanism for the generation of mass in particle physics.]